Coupled Oscillations of a Loaded String

$$-A_{r-1} + \left(2 - \frac{ma}{T}\omega^2\right)A_r - A_{r+1} = 0$$

taking $\omega_0^2 = \frac{T}{ma}$

$$\frac{A_{r-1} + A_{r+1}}{A_r} = \left(2 - \frac{ma}{T}\omega^2\right) = \left(2 - \frac{\omega^2}{\omega_0^2}\right)$$

$$= \frac{2\omega_0^2 - \omega^2}{\omega_0^2}$$

Let us express the amplitude of the r-th mass at the frequency ω_j as:

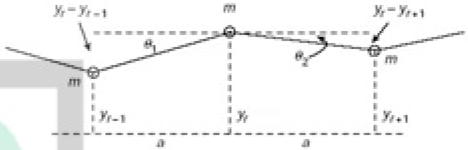
$$A_r = C \sin r\theta$$

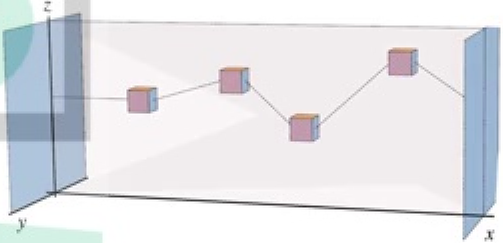
$$A_{r-1} = C \sin(r-1)\theta \; ; \; A_{r+1} = C \sin(r+1)\theta$$

$$\frac{A_{r-1} + A_{r+1}}{A_r} = \frac{C \sin(r-1)\theta + C \sin(r+1)\theta}{C \sin r\theta}$$

$$= \frac{\sin r\theta \cos\theta - \sin\theta \cos r\theta + \sin r\theta \cos\theta + \sin\theta \cos r\theta}{\sin r\theta}$$

$$= 2\cos\theta$$





$$A_r = C \sin r \cdot \theta$$

$$A_{r-1} = C \sin(r-1)\theta \; ; \; A_{r+1} = C \sin(r+1)\theta$$

To find θ_s at ω_s, we apply the boundary conditions:

$$A_0 = 0 \; ; \; A_{n+1} = 0$$

$$A_0 = C \sin 0 \cdot \theta = 0$$

$$A_{n+1} = C \sin(n+1)\theta = 0 \Rightarrow (n+1)\theta_s = s\pi$$

$$s = 1, 2, 3, 4, \ldots, n$$

$$\Rightarrow \theta_s = \frac{s\pi}{(n+1)}$$

$$A_{rs} = C_s \sin \frac{s\pi}{(n+1)} \cdot r$$