Physics Informed Neural Networks (PINNs)

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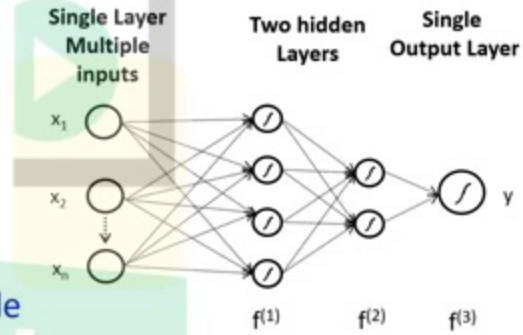
A Multi-Layer Perceptron (MLP)

- Input data is processed over multiple layers of neurons.
- Each layer 'i' of neuron is associated with implementing a certain function $f_i()$.
- Final output follows chaining of multiple functions, associated with each layer, given by

$$y = f^{(3)}\left(f^{(2)}\left(f^{1}(x)\right)\right)$$



Feedforward nature:
Information flows in one direction only,
i.e. from input to output

Loss function & Learning Algorithm

Loss function is a measure of deviation or inaccuracy, or cost we want to minimize

- **Choice of Loss function** depends on nature of problem, i.e. MSE (regression) or Cross Entropy (binary or multi-class classification problem)
- Given by **L(θ)**

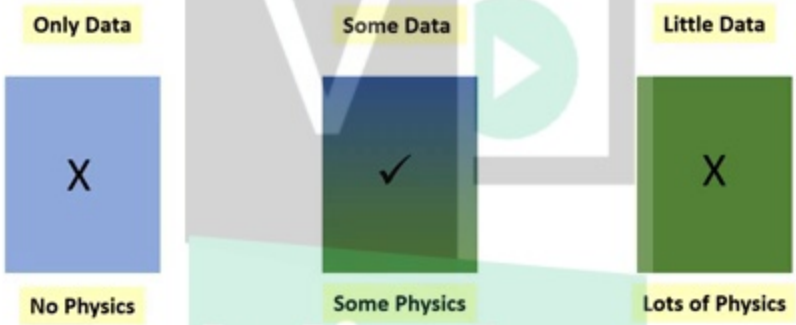
Learning algorithm refers to gradient descent technique used to adjust weights to minimize L(θ)

- $\theta' = \theta - \eta \nabla_\theta L(\theta)$, where, η is the learning rate.

ble of Physics & Data

Adapted from
G. Karniadakis talk

Three scenarios



PINNs address some data, some physics domain, where

- ❑ Some physics or governing equation is known, data is complicated, and noisy, or
- ❑ Don't have all boundary conditions to determine solution
- ❑ All of physics is not known.

Example Differential Equation

Consider the differential equation below

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0 \quad , y(0) = y0, y(1) = y1$$

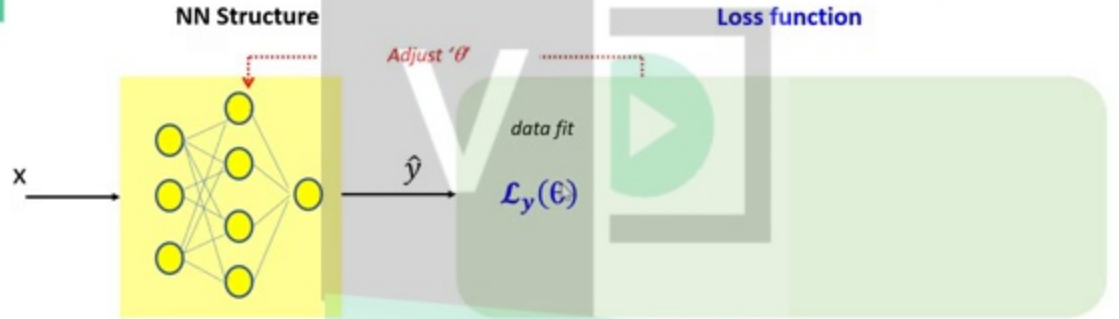
a second order linear differential equation with boundary conditions

- **Approaches to solve a differential equation**

1. Analytic solution (for simple equations like above)
2. Finite difference method

Loss Function Development

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$



Solving Burgers' Equation

M. Raissi, J. Comp. Phy., 2019