Information Visualization Team
Universität Magdeburg

Institute of Computer Graphics
and Knowledge Visualisation

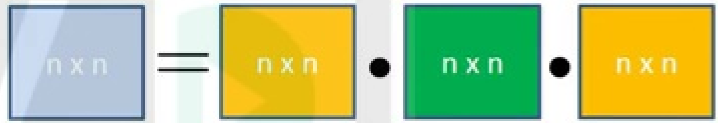
Data, Visualization, and Visual Analytics

- Part I: Basics in Math and Data Categories

TU Graz, SS 2017

Diagonalization

$$\mathbf{A} = \mathbf{X}\,\mathbf{D}\,\mathbf{X}^{-1}$$



$$\mathbf{A}\;\mathbf{X} = \underbrace{\mathbf{X}\;\mathbf{D}}_{\text{Scaling}}$$

$\mathbf{A}$ ...squared Matrix (n x n)

$\mathbf{D}$ ...Diagonal Matrix

Which vectors are scaled for $\mathbf{A}$?

Singular Value Decomposition

$$\underset{m\times n}{\mathbf{A}} = \mathbf{U}\underset{m\times n}{\mathbf{\Sigma}}\mathbf{V}^T$$

$\mathbf{U}_{m\times m}$ …Orthogonal Matrix $(\mathbf{U}^T = \mathbf{U}^{-1})$	$\mathbf{V}_{n\times n}$ …Orthogonal Matrix $(\mathbf{V}^T = \mathbf{V}^{-1})$

Singular Value Decomposition

$$\underset{m\times n}{\mathbf{A}} = \mathbf{U}\underset{m\times n}{\mathbf{\Sigma}}\mathbf{V}^T$$

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$$\overbrace{\mathbf{A}^T\mathbf{A}}^{\text{symmetric, pos. def.}} = (\mathbf{V}\mathbf{\Sigma}^T\underbrace{\mathbf{U}^T)(\mathbf{U}}_{\mathbf{I}}\mathbf{\Sigma}\mathbf{V}^T) = \mathbf{V}\underbrace{\mathbf{\Sigma}^T\mathbf{\Sigma}}_{\mathbf{D}\text{...Diagonal Matrix}}\mathbf{V}^T = \underbrace{\mathbf{V}\mathbf{D}\mathbf{V}^T}_{\text{Diagonalization}}$$

Since it is a diagonalization, it follows:

$\mathbf{V}$ = Matrix of Eigenvector of $\mathbf{A}^T\mathbf{A}$

Singular Value Decomposition

$$\underset{m\times n}{\mathbf{A}} = \mathbf{U}\underset{m\times n}{\boldsymbol{\Sigma}}\mathbf{V}^T$$

$\mathbf{U}_{m\times m}$ …Orthogonal Matrix $(\mathbf{U}^T = \mathbf{U}^{-1})$	$\mathbf{V}_{n\times n}$ …Orthogonal Matrix $(\mathbf{V}^T = \mathbf{V}^{-1})$

Singular Value Decomposition

$$\underset{m \times n}{\mathbf{A}} = \mathbf{U} \underset{m \times n}{\mathbf{\Sigma}} \mathbf{V}^T$$

$\mathbf{U}_{m \times m}$ ...Orthogonal Matrix ($\mathbf{U}^T = \mathbf{U}^{-1}$)	$\mathbf{V}_{n \times n}$ ...Orthogonal Matrix ($\mathbf{V}^T = \mathbf{V}^{-1}$)

- $\mathbf{U}$ = Matrix of Eigenvector of $\mathbf{A}\mathbf{A}^T$
- $\mathbf{\Sigma}$ ...Diagonal Matrix with Singular Values on diagonal.
  Singular Value σ_i of $\mathbf{A}$ equates to Eigenvalue λ_i of $\mathbf{A}^T\mathbf{A}$

  $$\sigma_i(\mathbf{A}) = \sqrt{\lambda_i(\mathbf{A}^T\mathbf{A})}$$
- $\mathbf{V}$ = Matrix of Eigenvector of $\mathbf{A}^T\mathbf{A}$

Solving linear Systems

m<n ..underconstrained

m...number of equations

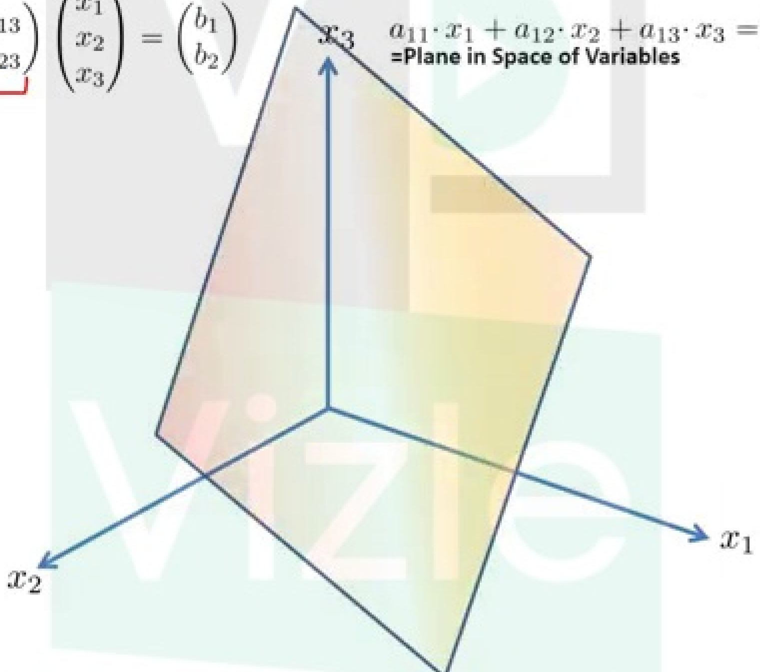
n...number of variables

$$\mathbf{A} \bullet \mathbf{x} = \mathbf{b}$$

$$m = 2 \left\{ \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}}_{n=3} \right. \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 = b_1$$

=Plane in Space of Variables



Solving linear Systems (Singular Value Decomposition)

-- Singulärwertzerlegung --

Solving Algorithms

m<n ..underconstrained

$$A\mathrm{x} = \mathrm{b}$$

$$U \cdot \Sigma \cdot V^T \cdot \mathrm{x} = \mathrm{b}$$

$$\Sigma \cdot (V^T \cdot \mathrm{x}) = U^T \cdot \mathrm{b}$$

$$\Sigma \cdot \mathrm{y} = U^T \cdot \mathrm{b}$$

Scaling

Substitution of y: $\mathrm{x} = V \cdot \mathrm{y}$

-SVD auch für m=n und m>n geeignet