Scheme & Solution of Model Question paper-1

Calculus and Differential Equations (21MAT11)

Module-1

1a) With usual notations prove that $\tan\phi = r\frac{d\theta}{dr}$

Soln: Explanation with figure & proof 2+4

Note: For proof one can refer previous video link provided in the description.

1b) Find the angle b/w the curves $r = a(1+\cos\theta)$ & $r = b(1-\cos\theta)$

is the required pedal equation.

2) Find the radius of curvature of the curve $x^3+y^3=3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.

Soln: Given $x^3+y^3=3axy$

Diff w.r.t $x \Rightarrow 3x^2+3y^2y_1 = 3a(xy_1+y)$

$$\Rightarrow y_1 = \frac{ay-x^2}{y^2-ax} \rightarrow (1)$$

$$y_1\left(\frac{3a}{2}, \frac{3a}{2}\right) = -1$$

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1c. If $x+y+z = u$, $y+z = uv$ & $z = uvw$

find $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$

Soln: Given $z = uvw \Rightarrow y = uv - uvw$ & $x = u - uv$

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix} = \begin{vmatrix} 1-v & -u & 0 \\ v-vw & u-uw & -uv \\ vw & uw & uv \end{vmatrix}$$

$$= (1-v)\{u^2v - u^2vw + u^2vw\} + u\{uv^2 - uv^2w + uv^2w\} + 0$$

$$= u^2v$$

from the original?

Soln: $T = t_2 + (t_1 - t_2)e^{-kt}$

where $t_1 = 80^\circ C$, $t_2 = 40^\circ C$, $T = 60^\circ C$, $T = 40\text{min}$?

$$T = 40 + 40e^{-kt}$$

$$60 = 40 + 40e^{-20k}$$

$$20 = 40e^{-20k}$$

$$e^{-20k} = \frac{20}{40} \Rightarrow e^{20k} = 2$$

$$20k = \log 2$$